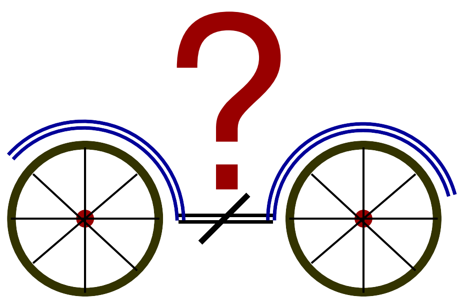
Coulomb Drag of Electrons for Bicyclists.

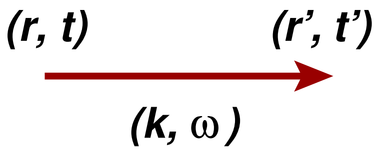
A. G. Yashenkin
PNPI and SPbSU



Outline.

- Constituents of the Theory.
- The Essence of the Coulomb Drag Effect.
- Detailing the Effect.
- Extensions and Exotics.
- Conclusion.

1. Constituents of the Theory: Diagramatics.

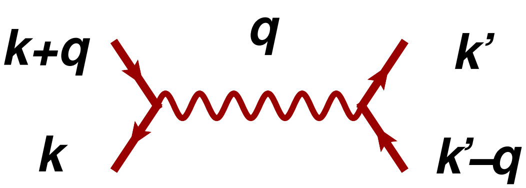


Electron Propagator

(Green Function)

$$G(r-r',t-t') = -i\,\langle\,\hat{T}\,(\,\psi_H(r,t)\psi^\dagger_H(r',t')\,)\,\rangle$$

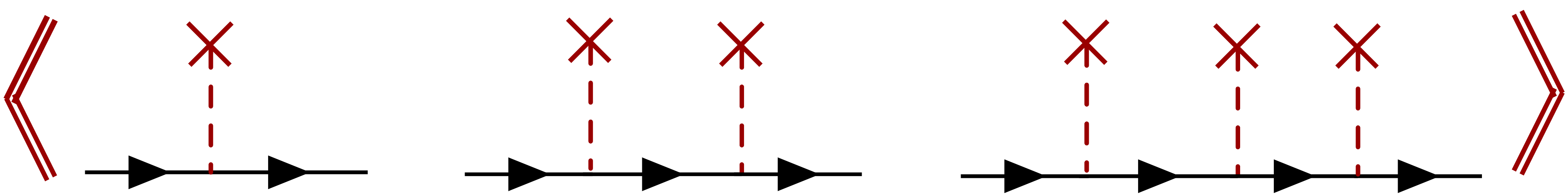
$$\psi_H(r,t) = e^{iHt}\,\psi(r)\,e^{-iHt}, \quad H = H_0 + H_{int}$$



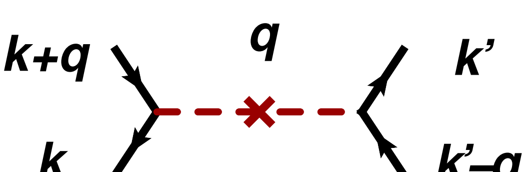
$$v_q = \frac{2\pi e^2}{q} \quad \longrightarrow \quad U_q(\omega) \propto (q+\varkappa)^{-1} \propto e^{-\varkappa q}/r$$

Coulomb Interaction

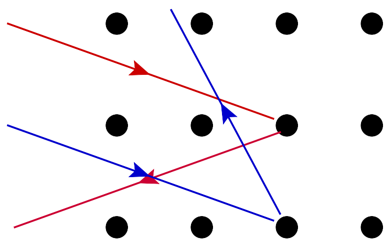
$$\propto (\omega-\omega_{pl})^{-1} \quad \omega > v_F q$$



Impurities Gaussian averaging over random configurations



1. Constituents of the Theory: Diffusion and Interference.

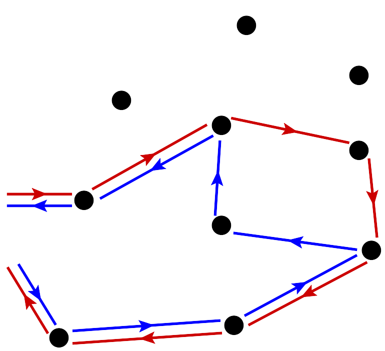


Bloch Theorem: $\varepsilon_0(\mathbf{k}) \longrightarrow \varepsilon(\mathbf{k})$

Classical Drude formula:

$$\sigma_D = \frac{n\,e^2\,\tau}{m}$$

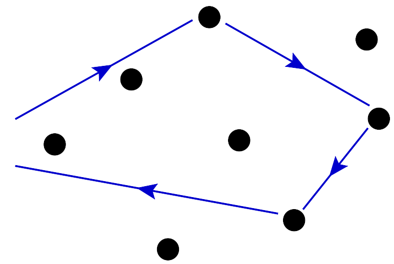
Why it works? $\lambda_F \ll l$



Ordinary Path $\longrightarrow$

$$\langle\, |A_1 + A_2|^2 \,\rangle_{\mathrm{traj}} \approx 2|A_1|^2$$

Classical Drude formula:

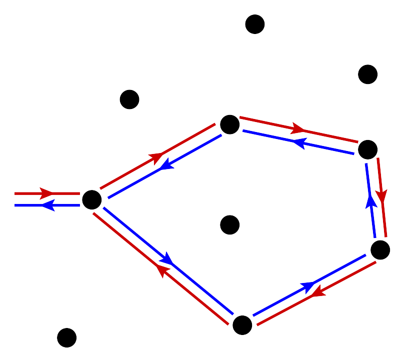


l – **Mean Free Path**

Einstein relation:

$$\sigma_D \propto e^2\, D\, \nu$$

D – Diffusion coefficient



Closed Path $\longrightarrow$

$$\langle\, |A_1 + A_2|^2 \,\rangle_{\mathrm{traj}} \approx 4|A_1|^2$$

Quantum Interference

1. Constituents of the Theory (Cont'd)

Diffuson (diffusion)

$$D \propto \frac{1}{Dq^2 - i\omega}$$

Cooperon (interference)

$$C \propto \frac{1}{DQ^2 - i\omega + \tau_\phi^{-1}}$$

Diffusive vs Ballistic:

$$l \ll l_\varphi$$

$\tau_\varphi \, (l_\varphi)$ – **Dephasing Time (Length)**

FTD: connecting susceptibilities and correlators. Kubo formula.

$$\mathrm{m} = \hat{\chi}\, \mathrm{h}, \qquad \hat{C}_{mm} = \langle \mathrm{m}, \mathrm{m} \rangle : \qquad \hat{\chi} \propto \hat{C}_{mm}$$

$$\mathrm{J} = \hat{\sigma}\, \mathrm{E}, \qquad \hat{C}_{JJ} = \langle \mathrm{J_x}, \mathrm{J_x} \rangle : \qquad \hat{\sigma} \propto \hat{C}_{JJ}$$

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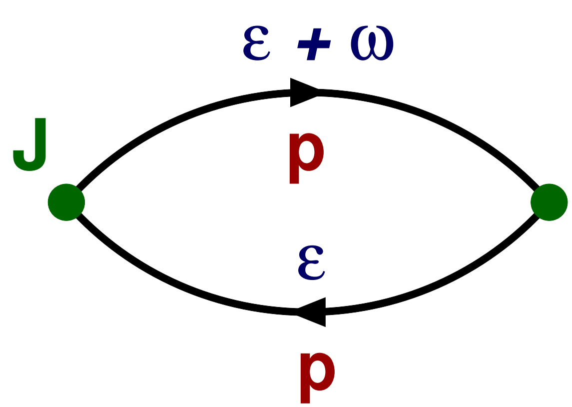

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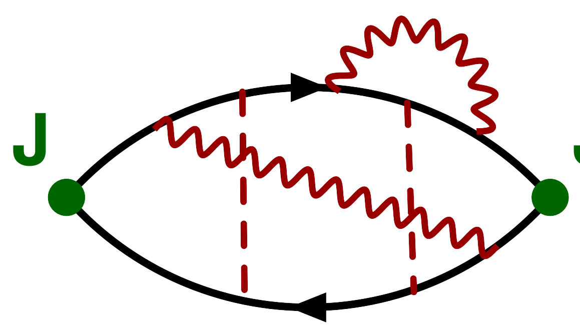
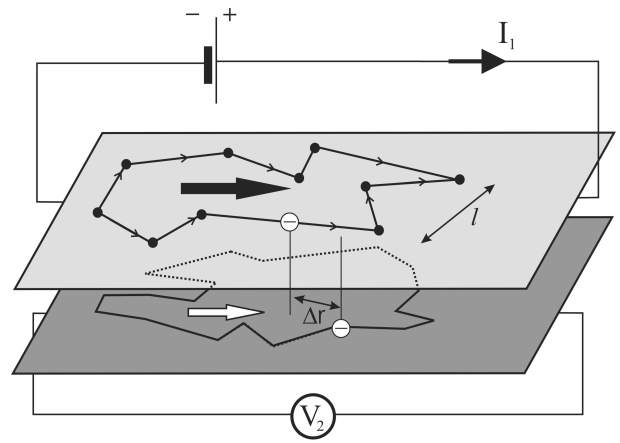


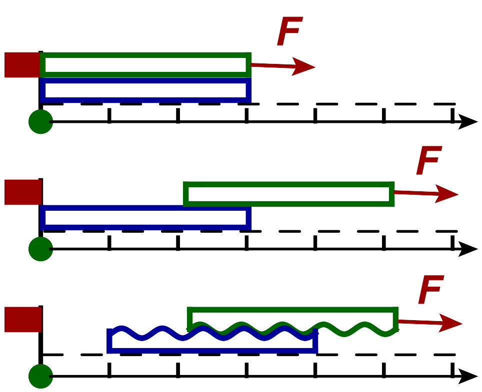
2. The Essence of the Coulomb Drag Effect.

Pogrebinskii '77, Gramila et al. '91, Eisenstein '92



Transresistivity $\rho_{21} = (V_2/J_1)_{J_2=0}$ $\rho_{21} = -\sigma_{21}/\det\hat{\sigma}$

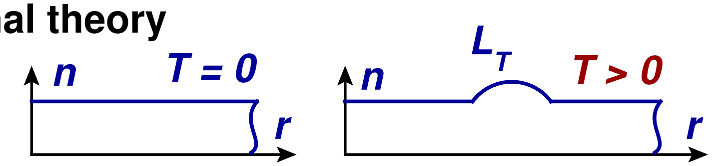
Conventional theory



sliding

roughnesses

friction ⟹ *dragging*

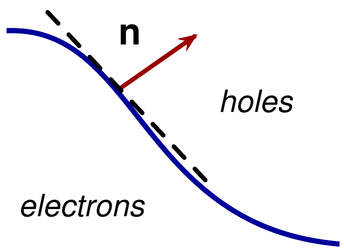


thermal density fluctuations

probabilities $\propto T/E_F$ & *independent*

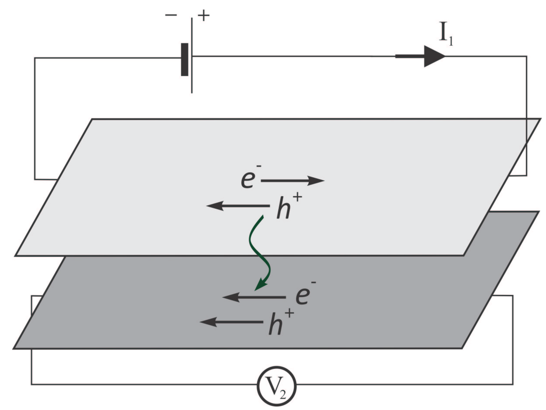
$\rho_{21} \propto (T/E_F)^2$

2. The Essence of the CDE (Cont'd).



$$\varepsilon_k = v_F (k - k_F) + $$
$$+ (2m)^{-1} (k - k_F)^2 + \dots$$

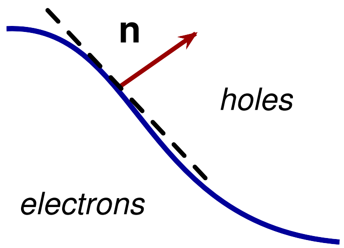
Particle–hole asymmetry



J_2 J_1

$$\sigma_{21} = i \ \omega^{-1} \ < J_2 \ , J_1 >_{\omega \to 0}$$

2. The Essence of the CDE (Cont'd).



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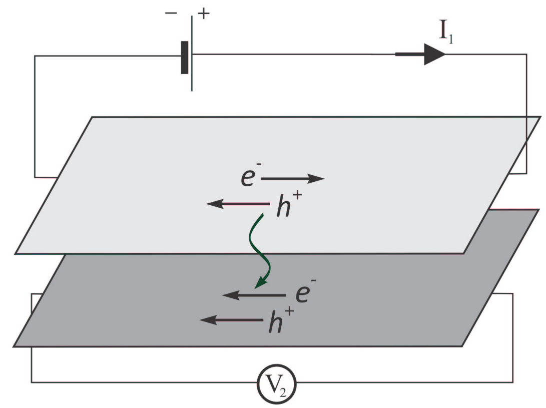
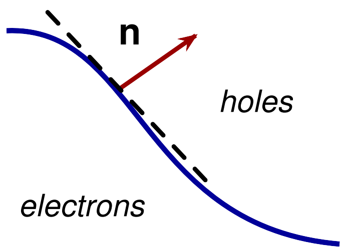


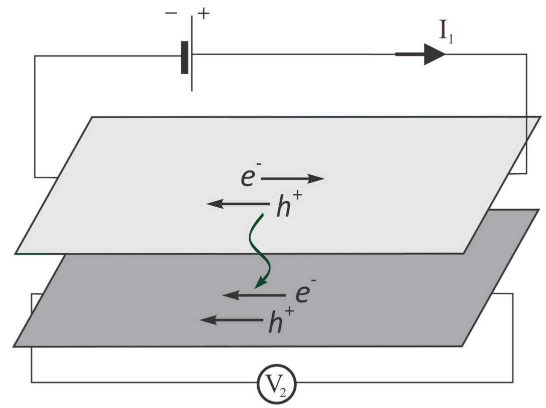
$$\sigma_{21} = i\ \omega^{-1}\ \langle J_2\ , J_1 \rangle_{\omega \to 0}$$

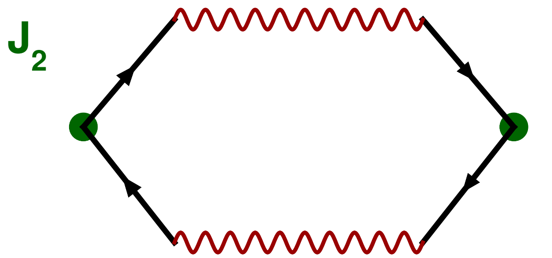
2. The Essence of the CDE (Cont'd).



$$\varepsilon_k = v_F\,(k - k_F) + (2m)^{-1}(k - k_F)^2 + \ldots$$

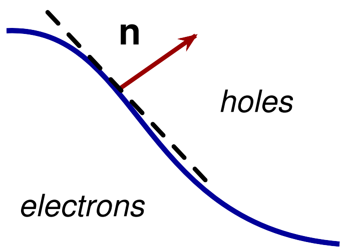
Particle–hole asymmetry





$$\sigma_{21} = i\ \omega^{-1} \langle J_2\,,\, J_1 \rangle_{\omega \to 0}$$

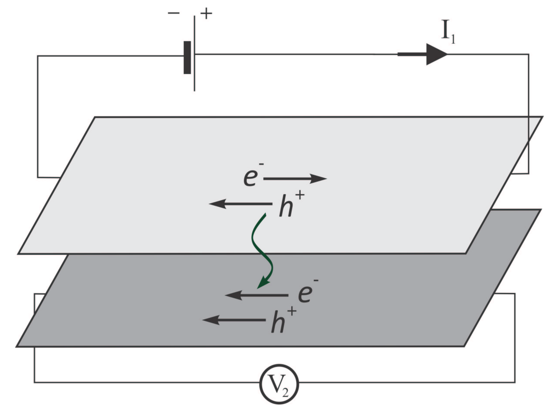
2. The Essence of the CDE (Cont'd).

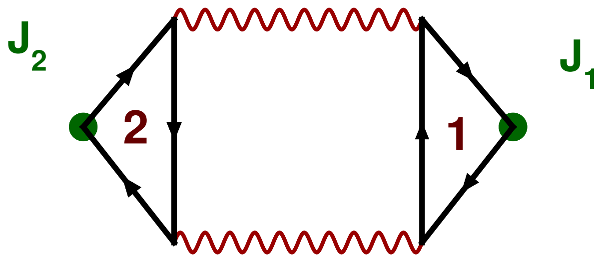


$\varepsilon_k = v_F (k - k_F) +$

$+ (2m)^{-1} (k - k_F)^2 + \ldots$

Particle–hole asymmetry





$$\sigma_{21} = i \ \omega^{-1} \ < J_2 \ , J_1 >_{\omega \to 0}$$

$$U_{21}(q,\omega) = \frac{\pi e^2}{\varkappa^2} \frac{q}{\sinh qd} \qquad \varkappa d \gg 1$$

$$\rho_{21} = \frac{1}{e^2} \frac{\pi^2 \zeta(3)}{16} \left(\frac{T}{E_F} \right)^2 \frac{1}{\varkappa^2 k_F^2 d^4}$$

2. The Essence of the CDE: Diff's and Sim's.

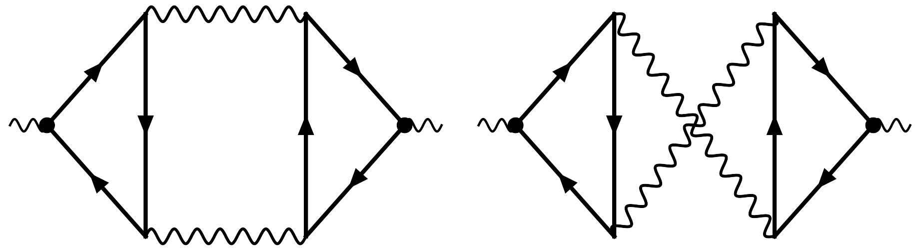
- **Phonon Drag:** phonons drag electrons when the temperature gradient is applied. FD manifests itself in thermopower.

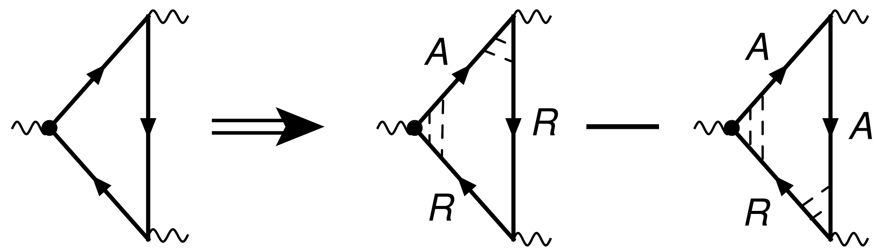
- **"Mixed" transport coefficients. Examples:**
  Hall coefficient $< \mathrm{J_x J_y} >$ is small in $(\omega_c \tau)$;
  Thermopower $< \mathrm{J_k J}_\epsilon >$ is small in (T/E_F) due to PHA;
  Transconductivity $< \mathrm{J_2 J_1} >$ is small in $(T/E_F)^2$ due to PHA.

- Interaction induced corrections to $< \mathrm{J_x J_x} >$.
  Aslamasov-Larkin contribution is smaller than Altshuler-Aronov one.
  However, now it is not a correction but the entire effect!

3. Detailing the Effect: Diffusion.

Typically, $(\omega\tau < 1,\ ql < 1,\ l_\varphi < l)$ $\rightarrow$ $T\tau < 1$. For CDE $qd < 1$.





$$U_{21}(q,\omega) = \frac{\pi e^2 q}{\varkappa^2 \sinh qd}\left(\frac{-i\omega + Dq^2}{Dq^2}\right)^2$$

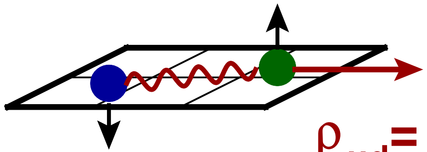
$$\rho_{21} = \frac{1}{e^2}\frac{\pi^2}{12}\left(\frac{T}{E_F}\right)^2 \frac{1}{(\varkappa d\, k_F l)^2}\ln\frac{T_0}{T}.$$

Thus, diffision dominates the drag only at vanishingly small T.

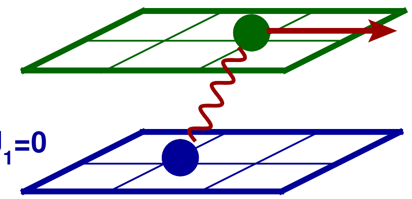
3. Detailing ...: Spin CDE and Correlated Impurities.

spin | **Coulomb drag** | **interlayer**

transresistivity

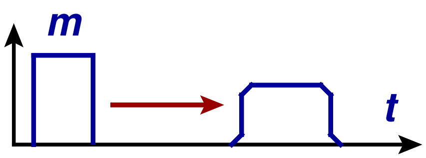


$$\rho_{ud} = (E_u / J_d)_{J_u=0}$$



$$\rho_{21} = (E_2 / J_1)_{J_1=0}$$

double quantum wells



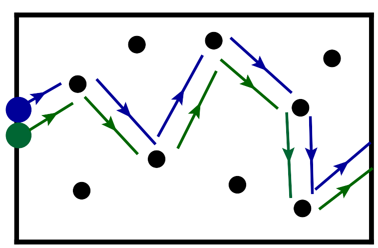
no spin flip!

$$D_c \propto D_s (1 - \rho_{ud} \, \sigma_D)$$

spin (charge) diffusion

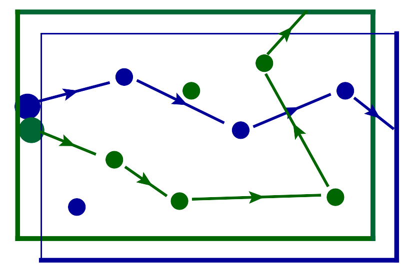
Gornyi et al. '99: experimental set up of correlated impurities in DQW

Impurity potential correlations. Quantum coherence.

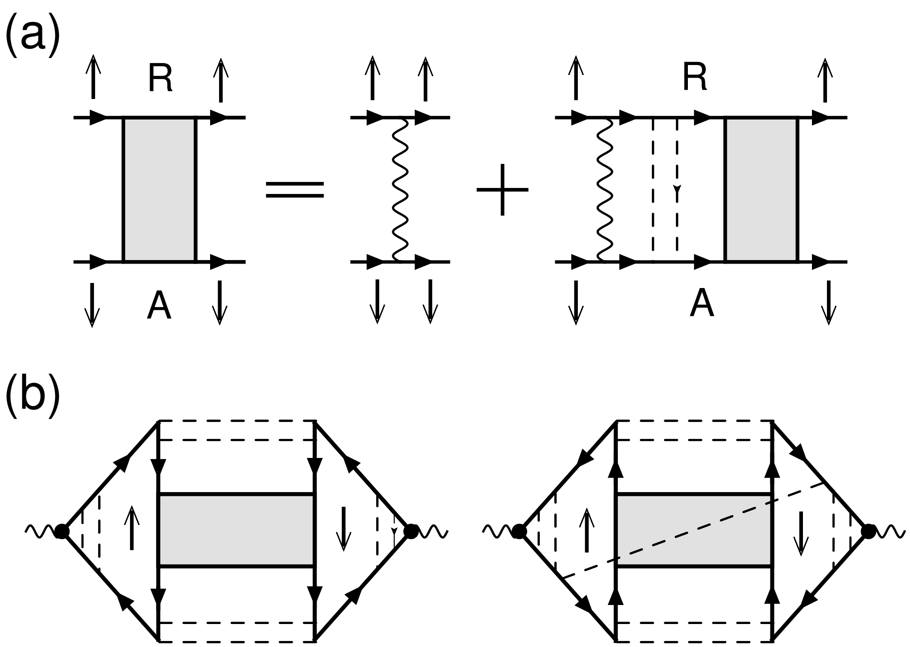


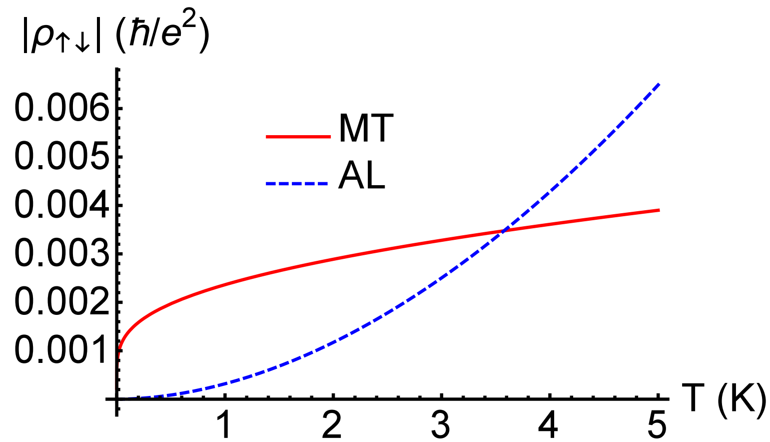
the same impurity potential for spin up and spin down

different potentials in layers



3. Detailing ...: Spin CDE and Correlated ... (Cont'd).





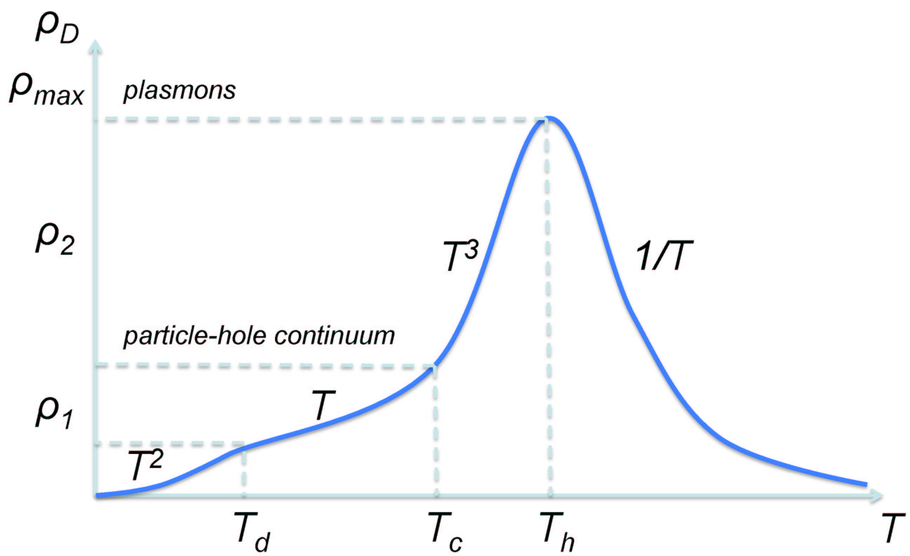
Anomalous Maki-Thomson processes. **Low-T enhancement.**

$$\rho_{21} = \frac{1}{e^2}\frac{\pi^4}{24}\frac{\ln(T\tau_g)}{(k_F d)^4\,(\varkappa l)^2}, \quad \tau_g^{-1} < T < \tau_{tr}$$

$$\rho_{21} = \frac{1}{e^2}\frac{\pi^4}{6}\frac{(T\tau_g)^2}{(k_F d)^4\,(\varkappa l)^2}, \quad T < \tau_g^{-1}$$

τ_g **measures the mismatch of random potentials in two layers.**

3. Detailing the Effect: Interaction and Temperature.



- The interlayer interaction is effectively static and screened. Fermi-liquid result $\boldsymbol{T^2}$.
- The interaction is retarded (dynamic) but overdamped. It excites the particle-hole pairs from the continuum. Linear-in-$\boldsymbol{T}$ behavior.
- $\boldsymbol{T^3}$ temperature dependence is due to interlayer plasmon modes.
- High-temperature regime wherein the density fluctuations could be treated hydrodynamically. Transresisitvity decays as $\boldsymbol{T^{-1}}$.

4. Extensions and Exotics.

CDE and SCDE between nanowires (including chiral LL's)

CDE of massless Dirac fermions (graphene and TI)

Interlayer macroscopic coherence (excitonic insulator, QH states)

CDE of composite fermions (FQHE)

Giant mesoscopic fluctuations of transconductance

DE of disordered bosons

Residual CDE (third-order)

CDE in low and intermediate magnetic fields

SCDE in presence of polarization and density mismatch

Phonon-mediated CDE

CDE between different fermionic systems

CDE in presence of tunneling briges

Conclusions.

- We overview the Coulomb Drag Effect in 2DEG.
- Over the past two decades, the Coulomb Drag Effect became a powerful tool for studying the intrinsic properties of interacting disordered low-dimensional electron systems.

Narozhny, Levchenko, Rev. Mod. Phys. '16

THANK YOU FOR YOUR PATIENCE !!!